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FINITE ELEMENT SIMULATION OF DYNAMIC BEHAVIOUR OF AN OPEN-ENDED CANTILEVER PIPE CONVEYING FLUID

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1. INTRODUCTION

A completely stagnant fluid-filled flexible pipe or a flexible pipe conveying fluid generally exhibits decrease in natural frequency [1]. Research studies for pipes conveying fluid are briefly reviewed by Blevins [1] and Chen [2]. In fact, a closed-form analytical solution is well known for the study of dynamic behaviour of pipes conveying fluid with different boundary conditions [1–3]. The governing equation of motion for free transverse vibration of a straight, tension-free pipe conveying fluid is given as [1]

$$EI\frac{\partial^4 Y}{\partial x^4} + \rho A v^2 \frac{\partial^2 Y}{\partial x^2} + 2\rho A v \frac{\partial^2 Y}{\partial x \partial t} + M\frac{\partial^2 Y}{\partial t^2} = 0, \tag{1}$$

where E is the modulus of elasticity of the pipe, I the area moment of inertia of the pipe, A the fluid flow area of the pipe, ρ the density of fluid, $M = m_p + \rho A$ the total mass per unit length, "m_p" is the pipe mass/length and v the flow velocity of fluid.

The first and fourth terms are stiffness and inertia terms whereas the second and third terms are fluid forces dependent on flow velocity. They are required to change the direction of flow in deformed/curved section of pipe during vibration and for rotation of fluid element respectively. The second term is equivalent to the axial compression term which increases with velocity. Hence, this term is responsible for frequency decrease with increase in flow velocity compared to the frequencies of the pipe completely filled with stagnant fluid. The third term is a mixed derivative that causes an asymmetric distortion in the classical mode shapes. Piping instability occurs at the critical velocity of the fluid in the pipe. Earlier studies [1, 2], based on experiments and analysis of pipes conveying fluid have focused on the instability of the pipes. Very few studies have been reported on experimental work towards the measurement of natural frequencies and mode shapes of the pipe conveying fluid. Further, the change in natural frequency is generally negligible compared to the pipe completely filled with stagnant fluid if the flow velocity is a small fraction of the critical velocity as the contribution by the second term of governing equation is negligible [1–3].

Analytical analysis can be carried out either by the above closed-form solution (equation (1)) or by FE analysis including the effects of the closed-form solution. Nowadays, the accepted practice is to use the FE model, which is simple for modelling even geometrically

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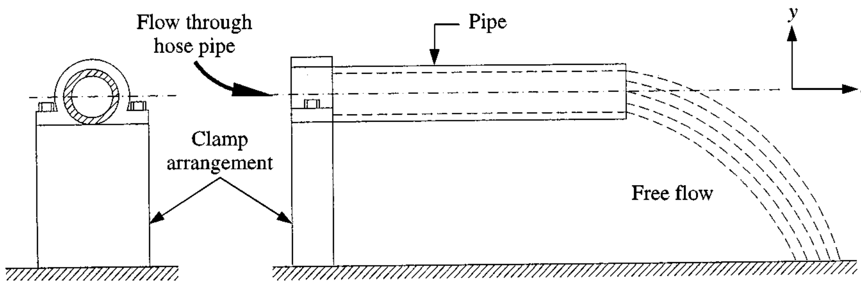

Figure 1. Experimental set-up.

complex structures, to solve engineering problems. In FE modelling, the decreasing in natural frequency of pipes conveying fluid is generally incorporated into the model as additional added mass to the pipe arising from the fluid contained in the pipe ("ρA", the fourth term of equation (1)) and decrease in pipe stiffness due to opposing action of flow (the second term of equation (1) as compressive axial force. The paper brings out the results of a modal experiment on an open-ended cantilever pipe conveying fluid and their analytical simulation using the FE method.

The schematic of a laboratory-scaled model of a cantilever pipe conveying fluid in shown in Figure 1. The pipe is made of aluminium of geometrical size–length 1010 mm, inner and outer diameters 22·85 and 19·65 mm. Young's modulus of elasticity (E) and mass per unit length (m_p) for the pipe are $70\,\mathrm{GN/m^2}$ and 0·028484 kg/m respectively. The modal experiments were carried out on the cantilever pipe at two water flow velocities: 0·55 and 1·1 m/s. The density of water (ρ) is $1000\,\mathrm{kg/m^3}$, so total mass per unit length (M) becomes equal to 0·5881 kg/m. The two flow velocities are small fractions of the critical velocity for the pipe which is estimated to be of the order of 350 m/s. These flows do not cause any instability in the pipe and the change in the natural frequencies is also expected to be negligible at these two flow velocities compared to the pipe filled with stagnant water. However, a significant change in natural frequencies was observed at these two flows. Hence, an attempt was made to simulate the changed dynamic behaviour of the pipe conveying fluid using FE analysis. In fact, no such analytical simulation of the experimental results was attempted for such a cantilever-type pipe conveying fluid in the earlier studies to the best of the authors' knowledge. The paper presents the analytical simulation using the FE method of the experimentally observed dynamic behaviour of the open-ended cantilever pipe conveying fluid.

2. EXPERIMENTAL DYNAMIC CHARACTERIZATION

The modal experiment on the cantilever pipe as shown in Figure 1 was conducted using the Impulse–Response method [4]. A small instrumented hammer was used for the excitation of the pipe. The force sensor (PCB 208A02) of the hammer and a tiny accelerometer (Entran PS-30A-2) were used for the measurement of the exciting force and the response of the pipe respectively. The impulse was given near the clamped location and response was obtained using accelerometers from a few location along the pipe. The measured time domain data were initially processed through a two-channel A and D 3525 FFT analyzer to obtain the averaged frequency response functions (FRFs). The modal parameters, i.e., natural frequencies and modal damping were then extracted using single-degree-of-freedom (s.d.o.f.) curve fit on the experimental FRFs through

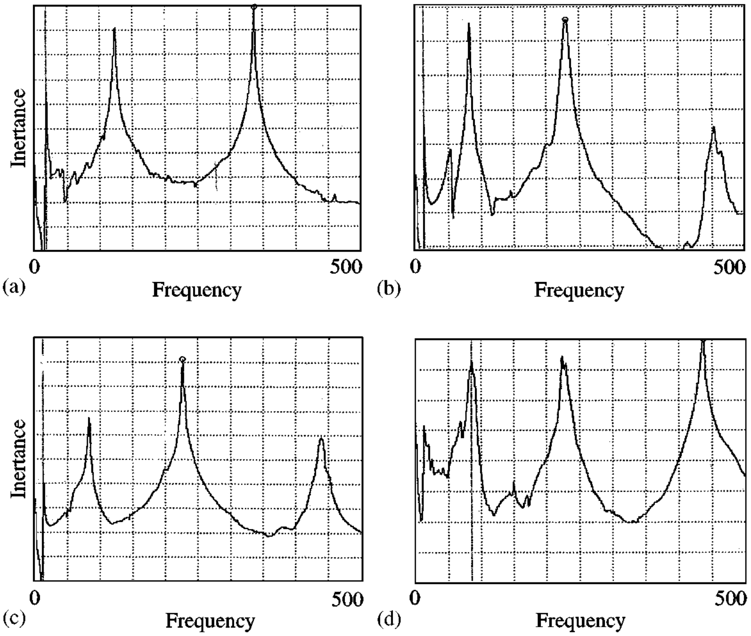

Figure 2. Typical experimental FRFs (inertance): (a) empty-pipe; (b) pipe filled with stagnant water; (c) pipe conveying water at 0·55 m/s; (d) pipe conveying water at 1·10 m/s.

computational program. The experiments were carried out under the following conditions: (1) Empty pipe—for estimation of the boundary conditions of the pipe in the model using FE model. (2) Pipe completely filled with stagnant water—for comparison with the dynamic characteristics of the pipe conveying fluid. (3) Pipe conveying water at a flow velocity of 0·55 m/s and (4) Pipe conveying water at a flow velocity of 1.1 m/s.

Typical experimental FRFs (inertance) are shown in Figure 2. Table 1 lists the experimentally identified natural frequencies for all the above four cases. Since the flows were only a small fraction of critical flow velocity, the reduction is stiffness is expected to be negligible for cases (3) and (4). Thus, no change in the natural frequency is expected for cases (2)–(4). However, it was observed that the natural frequencies change significantly from cases (2) to case (4). Further investigation was carried out by FE analysis.

3. FE ANALYSIS

A mathematical model for the cantilever pipe was made using the FE method [5]. The FE model of the pipe has been made using two-noded simple beam elements (two degrees of freedom at each node, i.e., bending moment and bending rotation) to simulate the cantilever beam-type modes of the pipe. Since, the pipe was mechanically clamped at one end, the clamped end of the pipe will have zero bending displacement and may have small rotation at that location. To simulate such a support condition, a spring element (one degree of freedom at each node) for the rotational restraint at the clamped location was provided in the FE model. The dynamic characterization of the empty pipe generally depends upon the physical and material properties and boundary conditions. The first two properties were known. The determination of boundary stiffness based on intuition/engineering judgement

TABLE 1

Experimental and analytical natural frequencies

Cases Flow rate		Case (1) Empty		Case (2) Static water		Case (3) Velocity = 0·55 m/s			Case (4) Velocity = 1·10 m/s		
		Exptl.	Analytl.	Exptl.	Analytl.	Exptl.	Analytl.		Exptl.	Analytl.	
							(1)	(2)		(1)	(3)
Natural frequency (Hz)	1st	18·43	18·70	13·12	13·02	12·44	13·00	12·54	11·81	12·94	11·93
	2nd	122·50	118·53	82·50	82·54	81·88	82·53	79·95	81·00	82·53	77·25
	3rd	336·56	334·57	229·69	232·96	226·25	232·96	226·51	222·81	232·96	220·46
	4th	—	659·86	455·00	459·44	440·75	459·44	448·35	435·93	459·44	438·94

Notes: (1), effect of flow velocity included; (2), effect of flow velocity + additional added mass (m) of 12 g; (3), effect of flow velocity + additional added mass (m) of 28 g.

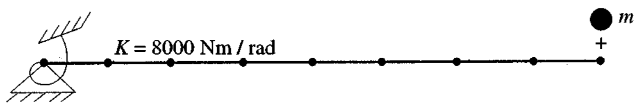

Figure 3. Typical FE model of the cantilever pipe. ● – Node point, m = additional mass simulating free jet flow.

may be erroneous. Hence, the best option is to use the experimental results of the empty pipe (case (1) of section 2) for the determination of the boundary stiffness of the pipe. The same was done. It was observed that a spring stiffness of 8000 m/rad was required to match the natural frequencies identified experimentally for the empty pipe. The FE model is now reflecting the actual behaviour of the pipe of the set-up. The FE model is shown in Figure 3.

For case (2), the mass of water contained in the pipe was only required to lump to the nodes of the above FE model. The so-computed natural frequencies match well with the experimental values as expected (Table 1). This further validates the FE model. Hence, the FE model can now be used for the simulation of cases 3 and 4. The reduction in the system stiffness due to opposing action of flow (second term of equation (1)) was also incorporated in the FE model. Such a system stiffness reduction, however, does not result in reasonable agreement with the experimental results for cases 3 and 4. The computed natural frequencies for cases (3) and (4) are almost the same as for case (2) as expected for such a small flow velocity, but significantly deviate from the experimental values. These computed frequencies are also listed in Table 1. Hence, the application of equation (1) is not straightforward for the case study under discussion. In fact, no such analytical estimation and their experimental verifications have been reported earlier.

One possible reason for change in the experimental frequencies could be the additional inertial force acting at the free end of the cantilever pipe accounting for the jet of water coming out of the pipe. The inclusion of such an effect in equation (1) based on intuition is a subjective one in the absence of experimental results. Now, since the experimental results are available, an attempt was made by simulating such an additional inertial force in the above FE model. This was done by adding an extra mass "m" of water as shown in Figure 3 at the free end of the pipe FE model. It was observed that the computed natural frequencies with additional fluid mass (m) values of 12 and 28 g are close to the experimentally obtained values at flow velocities of 0·55 and 1·1 m/s, respectively, as listed in Table 1 for cases (3) and (4). The computed values are well within $\pm$ 5% error compared with experimental values. Hence, the present study realized the need for addition inertia due to the free jet of flow in the analysis for correct dynamic characterization of the cantilever-type pipe conveying fluid.

4. CONCLUSION

As seen from the experimental and analytical study on an open-ended cantilever pipe conveying fluid, an additional inertia of fluid jet at the free end of the pipe is required to simulate the experimental natural frequency values. Hence, this additional inertia has to be accounted for in the governing equation of motion for the cantilever-type pipe conveying fluid as the boundary condition. An exact formulation for the additional mass of fluid to be added to the FE model without modal experiment requires a study on a large number of pipes of varying size and flow rate. Such a study is in progress.

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